Jet Fragmentation From Two Dimensional Field Theory

[FL, D. Kharzeev, arXiv:1111.0493]

Frashër Loshaj

Department of Physics & Astronomy
State University of New York at Stony Brook

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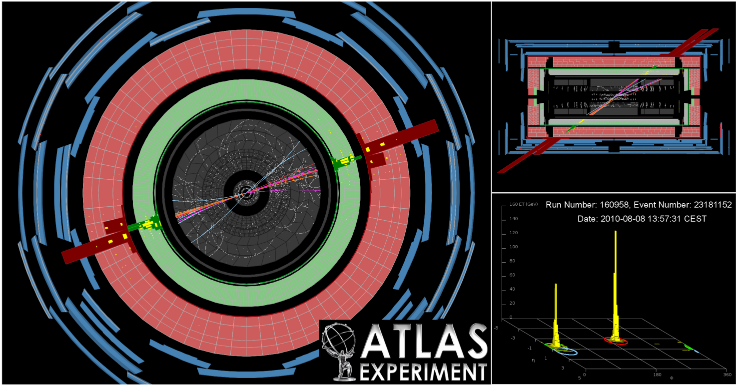
Jet events in particle detectors



- Example of a di-jet event observed at the LHC.

Jet fragmentation picture

Single-particle inclusive distribution (e.g. in $e^+e^- \to hX$)

$$F^h(x,s) = \sum_i \int_x^1 \frac{dz}{z} C_i(z, \alpha_s(s)) D_i^h(x/z, s)$$

$$s = q^2, \quad x = 2p_h \cdot q/q^2 = 2E_h/E_{cm}$$

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Fragmentation functions D_i^h evolve via DGLAP equations

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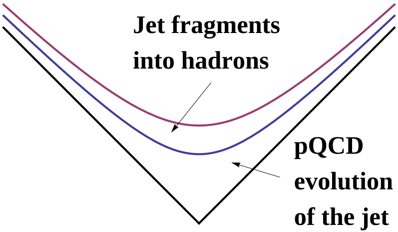

- At some scale $M^2 = Q_0^2 \sim 1-3$ GeV^2, $pQCD$ is not valid anymore.

Jet hadronization models

Jet hadronization models

- Some Ideas
  - Local parton hadron duality - flow of energy-momentum and flavor quantum numbers of hadrons should follow those of partons.
  - Universal low-scale - Assume that one can use $\alpha_s(q^2)$ even below $q^2 = Q_0^2 \sim 1$ GeV^2 to calculate Feynman diagrams.

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- Models
  - Cluster model - $q\bar{q}$ singlets have lower masses and form clusters, which in turn decay into pairs of hadrons.
  - String model - is based on the relativistic string stretched between initial quarks.

Dimensional reduction from QCD_4 to (1+1) field theory

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- Finally, an intuitive method to dimensional reduction was given in [Wong, 2009].

The Schwinger Model

The Schwinger model is QED in $1+1$ dimensions

Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - g\gamma^{\mu}A_{\mu} - m_q)\psi$$

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Dimensional analysis:

$$[A] = 0, \quad [\phi] = 0, \quad [\psi] = 1/2 \Rightarrow \boxed{[g] = 1}$$

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Interesting properties

- Dynamical Higgs Mechanism - Gauge field becomes massive via a Higgs mechanism induced by fermions in the theory.
- No free asymptotic charges exist in the theory - Charge screening.
- Linear confinement is also easily seen in the semi-classical treatment of the massive case.
- θ vacuum, similar to QCD_4.
- Using bosonization, it is shown that conserved currents have topological origin.

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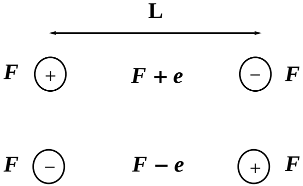

- Energy difference

$$\Delta E = \frac{1}{2}\int dx[F_{01}^2 - F^2] = \frac{1}{2}L[(F \pm g)^2 - F^2]$$

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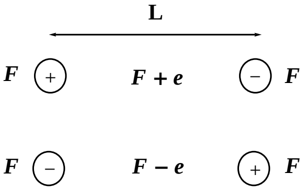

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- Physics is periodic in F, with period g!

Theta angle

$$\theta = \frac{2\pi F}{g}$$

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- Bosons and fermions different however, because of statistics.
- An explicit construction of a fermionic field out of a boson was given by [Mandelstam, 1975]

$$\begin{aligned}
\psi_L(x,t) &= \sqrt{\frac{c\mu}{2\pi}} : \exp\left(-i\sqrt{\pi}\left(\int_{-\infty}^{x} d\xi[\pi(\xi)+\phi(x)]\right)\right) : \\
\psi_R(x,t) &= \sqrt{\frac{c\mu}{2\pi}} : \exp\left(-i\sqrt{\pi}\left(\int_{-\infty}^{x} d\xi[\pi(\xi)-\phi(x)]\right)\right) : \\
\pi(x) &\equiv \dot{\phi}(x)
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- Using this relation between bosonic and fermionic fields, it is possible to verify the correct (anti)commutation relations.

Abelian Bosonization Rules

Operator	Fermionic	Bosonic
$J(z)$	$:\psi^\dagger\psi(z):$	$i\partial\phi(z)$
$\bar{J}(\bar{z})$	$:\tilde{\psi}^\dagger\tilde{\psi}(\bar{z}):$	$-i\bar{\partial}\phi(\bar{z})$
$T(z)$	$-\frac{1}{2}:[\psi^\dagger\partial\psi-\partial\psi^\dagger\psi]:$	$-\frac{1}{2}:\partial\phi\partial\phi(z):$
$\bar{T}(\bar{z})$	$-\frac{1}{2}:[\tilde{\psi}^\dagger\partial\tilde{\psi}-\partial\tilde{\psi}^\dagger\tilde{\psi}]:$	$-\frac{1}{2}:\bar{\partial}\phi\bar{\partial}\phi(\bar{z}):$
$\mathrm{fermion}_\mathrm{L}$	$\psi(z)$	$:\mathrm{e}^{i\phi(z)}:$
$\mathrm{fermion}_\mathrm{R}$	$\tilde{\psi}(\bar{z})$	$:\mathrm{e}^{i\phi(\bar{z})}:$
mass term	$\tilde{\psi}^\dagger(\bar{z})\psi(z)+\psi^\dagger(z)\tilde{\psi}(\bar{z})$	$\mu:\cos\hat{\phi}(z,\bar{z}):$

Bosonization in the "complex" formulation.

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Bosonization in the "complex" formulation.

- In Minkowski space-time we get

Maxwell current

$$j^\mu(x) =: \psi(x)\gamma^\mu\psi(x) := -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi$$

Bosonized QED_2

Massive QED_2

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Through correspondence

$$\begin{aligned} J^\mu(x) &= \bar{\psi}\gamma^\mu\psi = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi(x) \\ M^2 &= const.m_q\frac{g}{\sqrt{\pi}} \end{aligned}$$

Confinement versus screening

The string tension for a static case can be calculated, by adding an external source J^{μ}_{ext}, which enters in the Lagrangian as

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If we put an external pair with charges $\pm g$, separated by $2L$

$$J^{0}_{ext}(x) = \delta(z+L) - \delta(z-L)$$

It can be shown that

Potential

$$\begin{aligned} V(L) &= 2\pi^2 M^2 2L + \frac{g\sqrt{\pi}}{2}\left(1 - e^{-\frac{g}{\sqrt{\pi}}2L}\right) \\ M &\propto m_q g \end{aligned}$$

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- For $m_q = 0$, we get screening.
- Massive case, $m_q \neq 0$, shows linear confinement, with string tension $\sigma = 2\pi^2 M^2$.

Conserved currents

In massless QED ($m_q = 0$) two currents are conserved classically

Vector current

$$J_V^\mu = \bar{\psi}\gamma^\mu\psi$$

Axial Current

$$J_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

$$\partial_\mu J^\mu_{V,A} = 0$$

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We saw before that

$$J_V^\mu = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi$$

In $1+1$ dimensions $\gamma^\mu\gamma^5 = \epsilon^{\mu\nu}\gamma_\nu$, from where

$$\boxed{J_A^\mu = \frac{1}{\sqrt{\pi}}\partial^\mu\phi}$$

Useful relation

EOM of the gauge field, together with the bosonization formula for the vector current give us

$$\partial_\mu F^{\mu\nu} = gJ^\nu = -\frac{g}{\sqrt{\pi}}\epsilon^{\nu\alpha}\partial_\alpha\phi \Rightarrow \partial_1\left(F^{10} + \frac{g}{\sqrt{\pi}}\phi\right) = 0$$

If we require functions to vanish at $z \to \pm\infty$, we get

$$F^{10} = F_{01} = -\frac{g}{\sqrt{\pi}}\phi$$

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We will use this relation later to derive the anomaly equation.

Effective Lagrangian for ϕ

In $1+1$ dimensions field strength has only one component $F_{01} \equiv F$.
Bosonized Lagrangian can be written ($m_q = 0$)

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi A_\mu \\ &= \frac{1}{2}F^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{g}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu A_\mu \\ &= \frac{1}{2}F^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g}{\sqrt{\pi}}\phi F\end{aligned}$$

We can integrate F (e.g. choose the gauge $A_0 = 0$, Jacobian of $\int \mathcal{D}A_1 \to \int \mathcal{D}F_{01}$ doesn't depend on F) to get

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Effective Lagrangian

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$$

This is just a free massive scalar field, with mass $m = \frac{g}{\sqrt{\pi}}$.

Anomaly equation

Equation of motion for ϕ is just the Klein-Gordon equation

$$(\Box + \frac{g^2}{\pi})\phi = 0$$

Using bosonization relations

$$\partial_\mu J_A^\mu = \partial_\mu \left(\frac{1}{\sqrt{\pi}} \partial^\mu \phi \right) = \frac{1}{\sqrt{\pi}} \Box \phi$$

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$$\partial_\mu J_A^\mu = \frac{g}{\pi} F_{01} = \frac{g}{2\pi} \epsilon^{\mu\nu} F_{\mu\nu}$$

This is the two dimensional version of the well known anomaly equation in *QED* (Adler-Bell-Jackiw anomaly).

Axial charge

F_{01} is just the electric field E. Therefore

$$\partial_\mu J_A^\mu = \frac{g}{\pi} E$$

If we have a background electric field $E \neq 0$ then, using Gauss' theorem

$$\int dzdt\partial_\mu \; J_A^\mu = Q_A(t=\infty) - Q_A(t=-\infty) = N_R - N_L = \frac{g}{\pi} \int dzdt \; E(z,t)$$

where

$$Q_A = \int dz J_A^0$$

In other words

Axial charge

$$N_R - N_L = \frac{g}{\pi} \int dzdt \; E(z,t)$$

Adding a general external source

Consider a general external source $J^{\mu}_{ext}(x) = j^{\mu}_{ext}(z,t)$. In bosonized form can be written as

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Which gives

Equation of motion

$$(\Box + m^2)\phi(x) = -m^2\phi_{ext}(x)$$

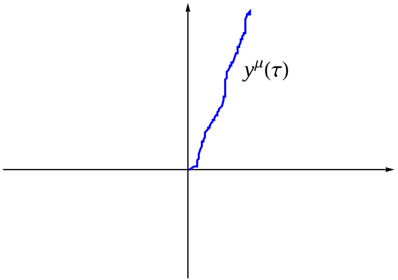
- Corresponds to a massive scalar field, coupled to a classical source.
- Coherent particle creation.

Constructing the external current

In general, we can construct a conserved current from

$$j^\mu(x) = \int d\tau \frac{dy^\mu(\tau)}{d\tau} \delta^{(2)}(x - y(\tau))$$

For a particle moving along the worldline $y^\mu(\tau)$



This construction gives a conserved current.

An example of particle creation

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Using bosonization relations we have

$$\phi_{ext}(x) = -\theta(t-z)\theta(t+z)$$

We therefore have to solve

$$(\Box + m^2)\phi = m^2\theta(t-z)\theta(t+z)$$

An example of particle creation

We consider the source [Casher, Kogut, Susskind, 1974]

$$J^0_{ext}(x) = \delta(z-t)\theta(z) - \delta(z+t)\theta(-z)$$

Using bosonization relations we have

$$\phi_{ext}(x) = -\theta(t-z)\theta(t+z)$$

We therefore have to solve

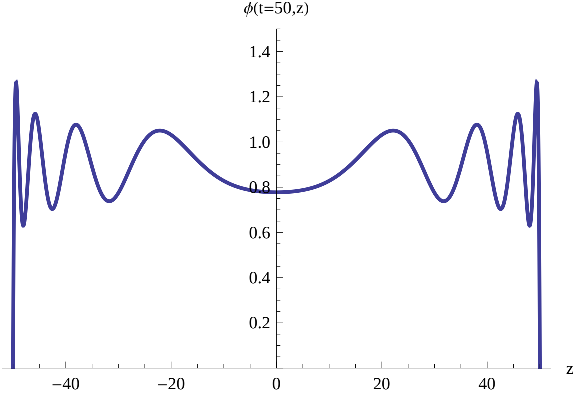
$$(\Box + m^2)\phi = m^2\theta(t-z)\theta(t+z)$$

The solution to the equation of motion is

$$\phi(x) = \theta(t+z)\theta(t-z)(1 - J_0(m\sqrt{x^2}))$$

where $x^2 = t^2 - z^2$.

An example of particle creation (cont'd)



(Anti-)Kinks correpond to (anti-)fermions.

Particle creation by a general source

In general, we can write

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Use

$$\phi_0(x) = \int \frac{dp}{2\pi}\frac{1}{(2E_p)^{1/2}}[a_p e^{-ip\cdot x} + a_p^\dagger e^{ip\cdot x}]$$

and

$$\Delta_R(x-y) = \int \frac{dp}{2\pi 2E_p}(e^{ip\cdot(x-y)} - e^{-ip\cdot(x-y)})\theta(x^0 - y^0)$$

Particle creation by a general source (cont'd)

$$\phi(x) = \int \frac{dp}{2\pi(2E_p)^{1/2}} \left[\left(a_p - \frac{i}{(2E_p)^{1/2}} \tilde{f}^*(p) \right) e^{-ipx} + \left(a_p^\dagger + \frac{i}{(2E_p)^{1/2}} \tilde{f}(p) \right) e^{ipx} \right]$$

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Since

$$H = \int \frac{dp}{2\pi} E_p \left[a_p^\dagger a_p + \frac{1}{2}[a_p, a_p^\dagger] \right] \Rightarrow \langle 0|H|0\rangle = \int \frac{dp}{2\pi} E_p \frac{|\tilde{f}(p)|^2}{2E_p}$$

Therefore

Hadron spectrum

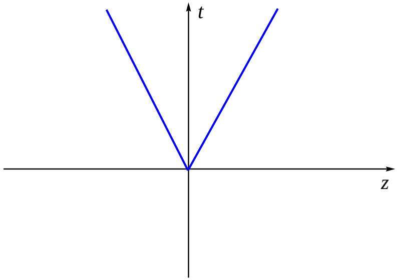
$$\frac{dN}{dp} \equiv \left\langle 0|a_p^\dagger a_p|0 \right\rangle = \frac{|\tilde{f}(p)|^2}{2E_p}$$

$|0\rangle$ is the free theory vacuum

Particle creation by a general source (cont'd)

More general charge density - quarks move with velocity v

$$j^0_{ext}(x) = \delta(z - vt)\theta(z) - \delta(z + vt)\theta(-z)$$



Velocity is calculated from

$$v = \frac{p_q}{E_q} = \frac{\sqrt{s}/2}{\sqrt{s/4 + Q_0^2}}$$

We can now calculate

$$\frac{dN}{dp} = 2\pi \frac{v^2 m^4}{E_p(E_p^2 - v^2 p^2)^2}$$

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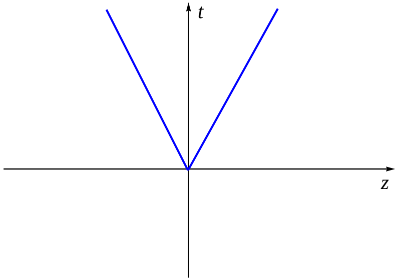

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We fix Q_0 by comparing our result to experimental data!

Jet variables

In $pQCD$, jets usually are described by rapidity y and variable z, defined as

$$
\begin{aligned}
y &= \frac{1}{2}\ln\frac{E_p+p}{E_p-p} = \ln\frac{E_p+p}{m} \\
z &= \frac{p}{E} = \frac{2p}{\sqrt{s}} \\
p &= m\sinh y \\
E_p &= m\cosh y
\end{aligned}
$$

where p, E_p and m are momentum, energy and mass of hadron. $E=\sqrt{s}/2$ is the jet energy.

Rapidity distribution

In bosonized QED_2, we calculated dN/dp. We now change variables to y and we get

$$\frac{dN}{dy} = 2\pi \frac{v^2}{(\cosh^2 y - v^2 \sinh^2 y)^2}$$

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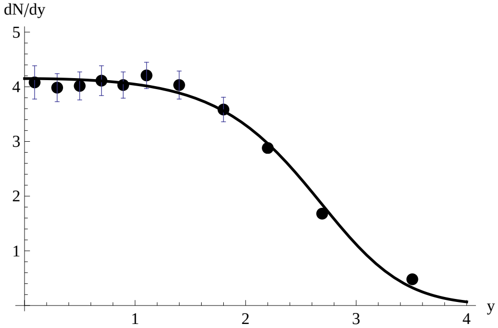

Comparison to experimental data [Aihara (TPC/Two Gamma Collaboration), 1988], for $\sqrt{s} = 29$ GeV

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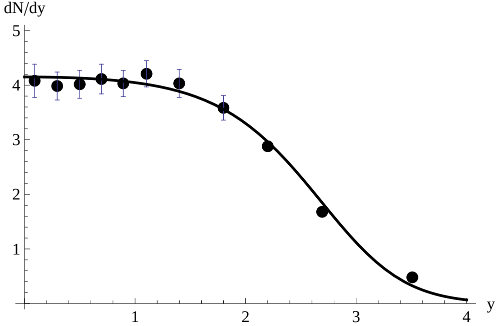
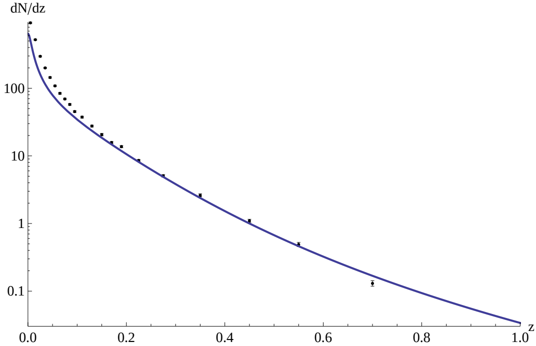


Comparison to experimental data [Aihara (TPC/Two Gamma Collaboration), 1988], for $\sqrt{s} = 29$ GeV

- Q_0 is fixed by above fit. We get $Q_0 \approx 1.8$ GeV.

Fragmentation functions

Fragmentation function for e^+e^- annihilation



Charged particle distribution for $\sqrt{s} = 201.7$ GeV, [Abbiendi (OPAL Collaboration), 2003].

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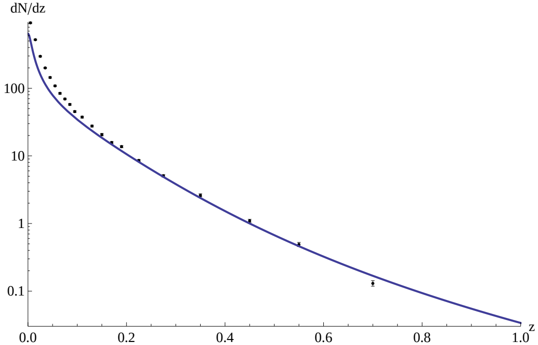

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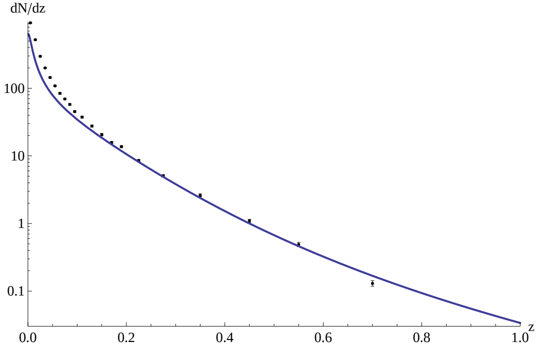
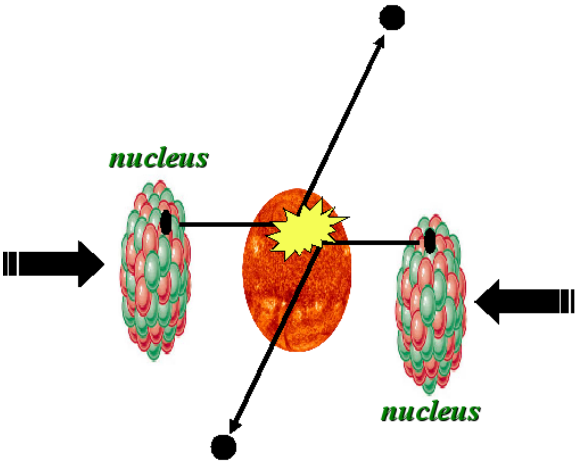


Charged particle distribution for $\sqrt{s} = 201.7$ GeV, [Abbiendi (OPAL Collaboration), 2003].

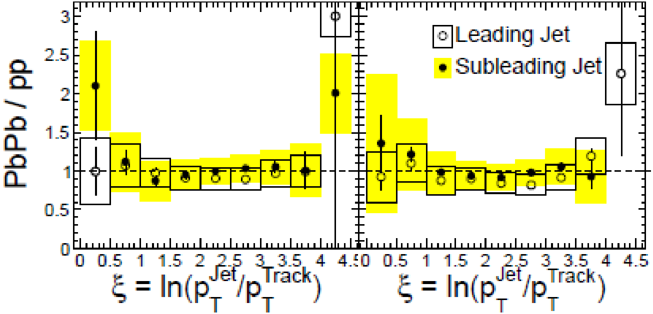
- Reasonable agreement with the data for $z > 0.1$.
- By fitting to data at different center of mass energies $m \simeq 0.6$ GeV.

Jets in Medium



Fragmentation scaling [Roland (CMS Collaboration), 2011]

Jet Fragmentation Function, PbPb≈pp



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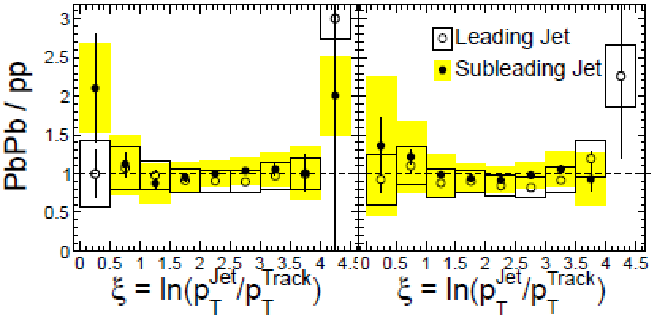
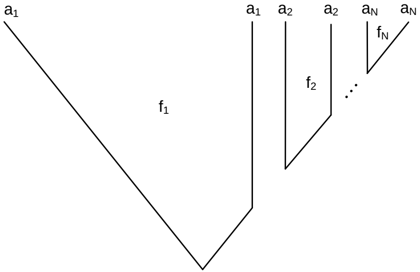


- Fragmentation functions are unmodified by the nuclear medium in heavy ion collisions.

Space-time representation

We treat scattering in medium in the following picture



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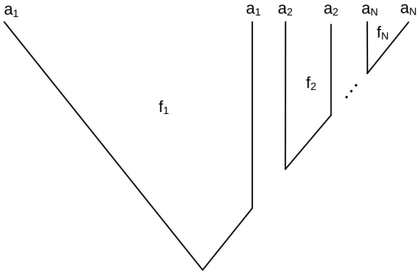

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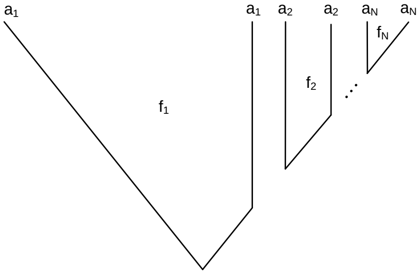


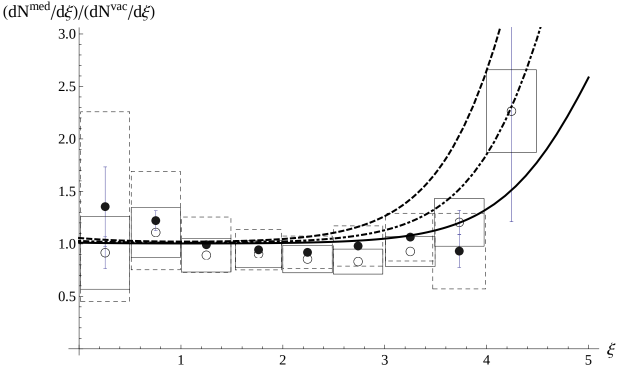
The medium is represented by static sources at points $z_i = vt_i$

Hadron spectrum is calculated from

$$\frac{dN^{resc}}{dp} = \frac{1}{2E_p}|\tilde{f}(p)|^2 = \frac{1}{2E_p}\left(|\tilde{f}_1(p)|^2 + \sum|\tilde{f}_2(p)|^2 + |\tilde{f}_3(p)|^2\right)$$

Contours live in different color sectors - large N picture, therefore there are no interference terms.

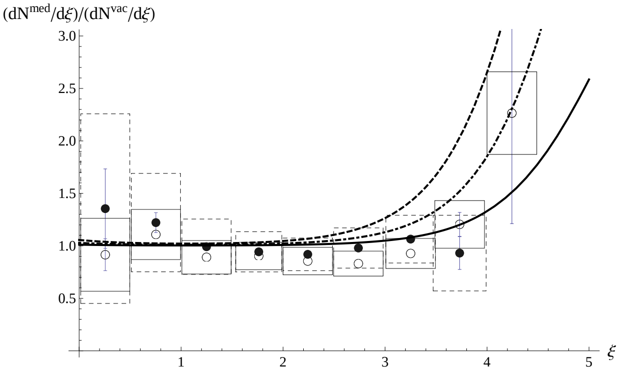
Fragmentation scaling (cont'd)



The length of the medium is fixed at 4 fm, the jet energy is $E_{jet} = 120$ GeV. Solid line: the first scattering occurs at $t_1 = 1$ fm (assumed thermalization time), and subsequent scatterings occur with time spacing of $\Delta t = 1/m = 0.3$ fm. Dashed line: double scattering with $t_1 = 2$ fm and $t_2 = 4$ fm ($\Delta t = 2$ fm). Dot-dashed line: four scatterings with $\Delta t = 1$ fm, $t_1 = 1$ fm. Open (filled) circles are for the leading (subleading) jet.

Fragmentation scaling (cont'd)

Counter intuitive - more scatterings give less emission!



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LPM Effect in Perturbation Theory [Review by Baier, Schiff, Zakharov, 2000]

Define formation time

$$t_{\text{form}} \simeq \frac{\omega}{k_\perp^2}$$

ω and $k_\perp$ are gluon energy and transerver momentum, $\omega >> k_\perp$ and $k_\perp \simeq \mu$.
And mean free path

$$\lambda = \frac{1}{\rho\sigma}$$

ρ is medium density, σ is the total scattering cross section.

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- When $t_{\text{form}} >> \lambda$ many scattering centers (N_{coh}) act as one

$$N_{\text{coh}} \simeq \sqrt{\frac{\omega}{\lambda\mu^2}} \equiv \sqrt{\frac{\omega}{E_{\text{LPM}}}}$$

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$$N_{\text{coh}} \simeq \sqrt{\frac{\omega}{\lambda\mu^2}} \equiv \sqrt{\frac{\omega}{E_{\text{LPM}}}}$$

- Energy spectrum can be estimated

$$\omega\frac{dI}{d\omega dz} \simeq \frac{\alpha_s}{\pi}N_c\sqrt{\frac{\mu^2}{\lambda}\frac{1}{\omega}}$$

Which is suppressed by a factor $\sqrt{E_{\text{LPM}}/\omega} \equiv \sqrt{\lambda\mu^2/\omega}$ compared to the Bethe-Heitler regime.

Non Perturbative LPM

From the picture shown earlier

$$
\begin{aligned}
\tilde{f}_1(p) &= \frac{-m^2 v\sqrt{\pi}}{E_p - vp}\left[\frac{2}{E_p + vp} - \frac{e^{i(E_p - vp)t_1}}{E_p}\right] \\
\tilde{f}_2(p) &= \frac{m^2 v\sqrt{\pi}}{E_p(E_p - vp)}\left[e^{i(E_p - vp)t_2} - e^{i(E_p - vp)t_1}\right] \\
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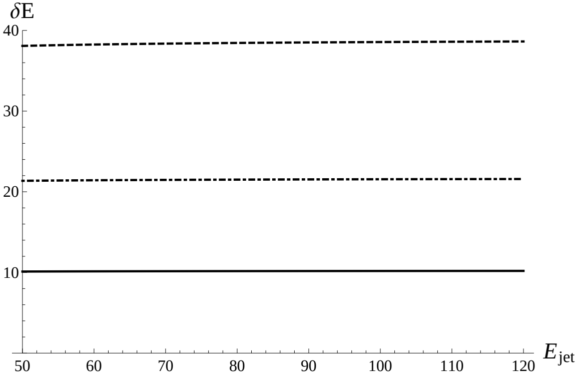
Contribution from $\tilde{f}_2(p)$ is suppressed for $t_2 - t_1 \equiv \Delta t$ (mean free path) small.

Energy Loss

Scaling is non trivial. We consider energy loss

$$\delta E = \int_{m_h}^{E_{jet}} dE_h E_h \left(\frac{dN^{med}}{dE_h} - \frac{dN^{vac}}{dE_h} \right)$$



Energy loss is mostly due to emission of soft particles.

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- Exact solution of this theory allows us to get a better understanding of non-perturbative and topological effects.
- May be a good starting point to study topological effects in high energy QCD.

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